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DIVISIBILITY BY SEVEN.

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The *Divisibility of Numbers*, as presented by different authors, embraces the conditions of divisibility by the numbers 2, 3, etc. up to 12, with the omission of the number 7. This omission leads us to inquire whether there is any general law for the divisibility of numbers by seven. A few of our text-books present some special truths in regard to this subject, among which are the following:

1. *A number is divisible by 7 when the unit term is one half or one ninth of the part on the left.* Thus 21, 42, 63, 126, and 91, 182, 273, etc.

2. *A number is divisible by 7 when the number expressed by the two right-hand terms is five times the part on the left, or one third of it.* Thus, 840, 525, 1995, and 3612, 903, 602, etc.

There are, however, some general laws for the divisibility by 7, which seem to have been overlooked by most writers on the theory of numbers, and which, though of no particular practical importance, are interesting in a scientific point of view. The first and least simple of these laws is as follows:

I. A number is divisible by 7 when the sum of once the first or units term, 3 times the second, 2 times the third, 6 times the fourth, 4 times the fifth, 5 times the sixth, once the seventh, 3 times the eighth, etc. is divisible by 7.

It will be seen that the series of multipliers is 1, 3, 2, 6, 4, 5. To illustrate the law, take the number 7935942, and we have for the sum of the multiples of the digits, $1 \times 2 + 3 \times 4 + 2 \times 9 + 6 \times 5 + 4 \times 3 + 5 \times 9 + 1 \times 7 = 126$, which is exactly divisible by 7; and if we divide the number itself by 7, we find there is also no remainder.

In this law we see that the second half of the series of multipliers 6, 4, 5, equals respectively 7 minus the first half, 1, 3, 2, hence, instead of adding the multiples of the second series, 6, 4, 5, we may subtract the respective multiples of the terms of the second numerical period by the first series 1, 3, 2, which will give rise to the following principle:

II. A number is divisible by 7 when the number arising from the sum of once the first term, 3 times the second, 2 times the third, minus the sum of the same multiples of the next three terms, plus the sum of the same multiples of the next three terms, etc., is divisible by 7.

It will be seen that the series of multipliers is 1, 3, 2, the first products being additive and the second products subtractive, etc.; the multiples of the terms of the *odd* numerical periods being additive and of the *even* periods subtractive. If we take the number 5439728, we have $1 \times 8 + 3 \times 2 + 2 \times 7 - 1 \times 9 - 3 \times 3 - 2 \times 4 + 1 \times 5 = 7$, which is divisible by 7; and upon trial we find the number is also exactly divisible by 7. This second principle may be more simply stated thus:

II_a A number is divisible by 7 when the sum of the multiples expressed by the numbers 1, 3, 2, of the terms of the odd numerical periods, minus the sum of the same multiples of the terms of the even numerical periods, is divisible by 7.

Now, if we add exact multiples of 7 to the multiples of the terms which are united in the test of divisibility, it will not change the remainder. Thus, taking the number 5439728, if we add 7×2 to 3×2 , we have 10×2 or 20, and adding 98×7 to 2×7 , we have 100×7 or 700; hence we may use, in place of $1 \times 8 + 3 \times 2 + 2 \times 7$, the sum of $8 + 20 + 700$, or 728, the first numerical period; and in the same way it may be shown that we may use the second period, subtracting in the last, etc. Hence from principle II. we may derive the following principle:

III. A number is divisible by 7 when the sum of the odd numerical periods, minus the sum of the even numerical periods, is divisible by 7.

To illustrate this principle, take the number 5,643,378,762; we have for the sum of the odd numerical periods $762 + 643 = 1405$; for the sum of the even periods $378 + 5 = 383$; the difference is 1022, which is exactly divisible by 7; and if we divide the number itself by 7, we find that there is also no remainder.

Applying the same reasoning to Principle I., by which we derived Principle III. from Principle II., we obtain the following Principle:

IV. A number is divisible by 7 when the sum of the double numerical periods is divisible by 7.

Thus in 5,643,378,762, the sum $378,762 + 5,643$, or 384405, is divisible by 7, and the number is also divisible by 7.

These principles may be put in the more general form of determining the remainder arising from dividing by 7. Thus the third principle may be stated as follows:

V. *Any number divided by 7 leaves the same remainder as the difference of the sums of the odd numerical periods and the even numerical periods divided by 7.* If the sum of the even periods is the greater we subtract the remainder from 7 for the true remainder, as in the similar case of the principle for 11.

The second, third, and fourth principles, as here presented, are derived from the first, though they may be demonstrated independently. The arithmetical demonstration of Prin. V. leads to a still more general law of divisibility, since the factor 1001, which will be seen to be the basis of the demonstration, is the product of 7, 11, and 13. The principle is thus seen to be true for 11 and 13 and the most general form of statement is as follows:

VI. *Any number, divided by 7, 11, or 13, leaves the same remainder as is obtained when the sum of the odd numerical periods minus the sum of the even numerical periods is divided by these numbers.*

From this we may immediately derive the law of exact divisibility by 7, 11, and 13, namely:

VII. *Any number is divisible by 7, 11, and 13 when the difference between the sums of the odd and even numerical periods is divisible by these numbers.*

Subsequent to the discovery of Prin. II., I learned that Prof. Elliott had employed the same property as early as 1846; whether it was known previously to this date has not been ascertained. I have also recently noticed that Prin. III. is given by one or two writers, but with whom it originated I am not aware. The other two laws, so far as I can learn, have not been previously published.

HISTORICAL SKETCH OF AMERICAN MATHEMATICAL PERIODICALS.

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From the settlement of the American Colonies to the commencement of the present century, but little attention was paid to Science and especially to mathematical Science. Many of the early settlers of the Colonies, and particularly the Clergy, were among the most eminent classical scholars in the world. Many of them had carried off the palm of victory at Oxford and Cambridge in England. Latin and Greek, and in some instances Hebrew, were as familiar to them as their native tongue. Some of them had even mastered the Chaldee, Syriac and Arabic languages. Theological speculation was indulged in to a great extent. In mathematics, on the other hand,